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Research Paper

Retrieving the Hidden Geometric Secret in the design of the *Naghsh-e- Jahan Square* based on Congruent Rotated Squares

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Abstract
One of the most famous historical squares of Iran is the Naghsh-e Jahan Complex, of which many studies have been done about its features and its effects on the urban space of Isfahan. The measurement of the geometric system governing the physical structure of this complex is considered an important part, of which no comprehensive research has ever been conducted. Accordingly, this study, aiming at analyzing the impact of the geometric organization and its order on the structure of the Square, and in particular on the location of its multiple accesses, seeks to evaluate this topic by using the relative measuring system based on the geometric rules as a process for the regularization of the space. The relative measurement system is one of the first ones, resulting in the application of geometry to the physical structure of the city and architecture. Its valuable benefits can be the application of dimensions and sizes with the identified coefficients in the cited plan. The present study is classified in the category of historical research, and descriptive and exploratory methods have been used in the historical research. Also, research on library data has provided the basis for analyzing the geometric structure of the Square and its dependent buildings. The research question can be formulated as follows: What is the relationship between the location of multiple entrances to the Naghsh-e Jahan square and its geometric system? The research findings indicate that the structure of the Naghsh-e Jahan Complex is following a geometric system with relative lengths based on the square shape, so the accesses location and the organization of the peripheral elements have all been subject to the mentioned system. Also, the results emphasize the benefit of a continuous process in planning the structure of the recent urban space, including its components, and considering it based on rational and inferential thinking, which, according to the geometric rules, is forming the hidden and revealed relations between the elements of the Safavid city.

Keywords: *Naghsh-e jahan square, Safavid era, Formation of the urban landmark, Geometric proportions, World cultural heritage retrieving, Golden ratio.*

1. INTRODUCTION

Urban sites in Isfahan are based on Euclidean geometry, with the formation of a complete geometric shape. One of the most prominent sites in Isfahan is *Naghsh-e-Jahan square*[1], which represents a successful attempt to harmonize nature with a geometric system to achieve unity between the natural and the artificial environments as a whole (Ahari, 2006). This square, built to respond to the urban needs of the Safavid Empire, revealed the main elements of the power of the empire in the form of a unique structure (Jafari Jabali & Shahabi Nejad, 2016, p. 227). A controversy about the first settlement of this square and its foundation dates its construction back to the *Safavid era*[2] and locates it in the *Chehel-Sotoun*

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[1] A historical square in the city of Isfahan.
[2] It was one of the most significant ruling dynasties of Iran from 1501 to 1736 A.D.

garden[1]. *The* current square, with a regular geometric plan and a rectangular shape, and with a north-south axis was constructed under the command of *Shah Abbas I*[2] or *Shah Ismaeil*[3] influenced by the *square of Hasan Padeshah (Saheb Abad) of Tabriz*[4] and *Ali-Qapu square (Saadat) of Qazvin*[5]. It has an area of 510 meters in length and 160 m in width (Habibi & Ahari, 2012, p. 47; Rafiee Mehrabadi, 1973, p. 20 & 287).

Due to its spatial relations, especially its combination method with urban architectural elements such as the government palace, mosque, and bazaar, this square is the product of special rules applied to urban design, with a system symbolizing the majesty, authority, and art of the *Safavid Empire*[6]. The effect of the square on the formation of *Safavid Isfahan* and its pre-designed structure is the application of special design rules and techniques to respond to the need to create a completely coordinated system. Also, the dominance of the geometric logic for governing the buildings and the cities in Islamic architecture, especially from the *Timurid period*[7]*on*, is the result of the use of a system based on the multiplication of the main forms, such as the equilateral triangle, the square, and the circle. This structure was realized through simple geometric rules based on the system of interconnection between the components, independent of numerical calculations and mathematical arguments (Najiboglu, 2010, p. 58 & 59). Therefore, the method of combining and adjusting the geometric and spatial space of the square and its internal and surrounding elements has led to its "readability" and "coherence" which is the expression of the geometric structures in Islamic art. The present article, based on the rules of applied geometry, using the historical method and library data, aims at understanding the geometric organization of the *Naghsh-e-Jahan square* and also at determining the relations of the square with the adjacent buildings on the one hand and with the system of accesses (the main entrances) on the other hand. To achieve this, the present study will analyze the square using the relative measurement system[8] as one of the most widely used methods in the urban/architectural structures of this country (Al-Saeed & Parman, 2010; Moulavi, 1991, summer, 2002).

2. REVIEW OF THE LITERATURE

Numerous studies have been performed on the *Naghsh-e-Jahan square*. Some studies have reviewed the genesis of this space and its historical evolution. They investigated the development of the square from its formation up to the most recent periods, with special emphasis on the documentation of the square changes (Dahar & Alipour, 2013; Shahabi Nejad, Abuei, & Ghaleh Noei, 2016, February; Shahabi Nejad, Abuei, Ghaleh Noei, & Emami, 2014, spring and summer; Utaberta, Mamamni, Surat, Che-Ani, & Abdullah, 2012).

In some of the studies, the process of evolution and the basis of the shape of the *Naghsh-e-Jahan square* in *Isfahan* have been investigated; which has been done by evaluating the formation process of the squares that were built in the prior capitals of the *Safavid period* (*Tabriz* and *Qazvin*). These researchers have claimed that the overall form structure of the *Naqsh-e Jahan complex* was similar to the previous models -such as *Sahibabad Square* in its heyday and glory (*Shah Tahmasb Safavid period*) - in such a way that the mentioned square can be considered a basic model for urban spaces with different functions (governmental-political, social-cultural, religious and sports) (Dizani, 2014; Hojat, Balali Oskouyi, & Yazdani, 2019). In another article, the influence of the urban space of the *Naqsh-e Jahan Square* in *Isfahan* on traditional *Iranian* urban spaces and *European* urban elements and squares in the *Renaissance period* has been investigated. In the mentioned research, the physical structure of the *Naqsh-e Jahan Square complex* in *Isfahan* is considered to be influenced by the urban spaces of the historical period before the *Safavid period*, especially the *Timurid period* in *Iran* and the

[1] Chehel-Sotoun Garden is one of the landmarks of Isfahan province and it is an example of a royal garden that was built during the Safavi era.

[2] He was the 5th Safavid Shah (king) of Iran, from 1588 to 1629 A.D.

[3] He was the founder of the Safavid dynasty of Iran, from 1501 to 1524 A.D.

[4] Saheb Abad Garden & its square of Tabriz, are one of the most important architectural building of the Aq-qoyunlu era (1373- 1501 A.D.).

[5] It is the important square of the Safavid Era in Qazvin city.

[6] The Safavid Empire, was one of the greatest Iranian empires after the 7th-century Muslim conquest of Persia, ruled from 1501 to 1736 by the Safavid dynasty.

[7] Timurid dynasty, (fl. 15th–16th Centuries CE), the dynasty of Turkic-Mongol origin descended from the conqueror Timur (Tamerlane).

[8] One of the results of applied geometry in the physical structure of buildings and the city is the appearance of the relative measurement system (the lengths are determined by a certain coefficient relative to a base length). This method has been the first method of measurement, one of the important benefits of relative systems is dimensions and sizes that are used with known coefficients. These common ratios have been included: 1, 2, 3, 1/2, 1/3, √2, √3, √2/2, √2/3, √3/3 and √5 + ½ (golden ratio), that were developed by dividing the base circle into 4, 5, and 6 parts. In this system, the plan was transmitted and changed scale, using lines, arcs, and circles. Also, because of not using measuring instruments, errors are very small (Moulavi, 2002, p. 16 & 17).

Renaissance period in *Europe* (Rahmani, Nasiri Kia, Kazemzade Raef, & Mir Derikondi, 2022).

Some studies were also conducted on the layout of the elements in the square and the process of their development by evaluating and comparing several written sources (Mojtahedzadeh, 2013-06-26; Radhamadi, Tehrani, & Aboei, 2011). In another study, the parameters of the coexistence of inside and outside of the square, and their role in the development of urban identity and architecture of the recent collection were measured. This study reported that the attitude changed in the relationship inside and outside and that the significance of the role of the threshold (semantic and functional) is the product of the prevalence of *Isfahan-style architecture* in the *Safavid period* (Mahboobi, Mokhtabad, Etesam, & Attarabbasi, 2018).

Some other studies have been performed to measure the form of the square. In this regard, it can be pointed out a study in which the historical center of *Islamic cities* such as *Isfahan* is considered a labyrinth to achieve different expressions for urban space. In this article, the physical characteristics of the *Naghsh-e-Jahan square* are detailed based on the sequential discovery of urban spaces (Falahat, 2018). Another study compared the features of the square with similar spaces, like other *Safavid squares* and *Sheikh Safi's tomb*[1], and emphasized the role of geometry, symmetry, hierarchy, and confinement in the design of the *Naghsh-e-Jahan square* (Aghabozorg & Motadayyen, 2015). In other studies, the role of the *Naghsh-e-Jahan square* as a key structure in the urban space of *Isfahan* was introduced using form analysis. They reported that the form of the *Naghsh-e-Jahan square* in the city is in accordance with sustainable social and environmental principles (Ahmadi, Asadpour, Pouryoosef Zadeh Najafabadi, & Azari, 2011; Rahbar & Ansari, 2016).

Analyzing the previous studies and comparing them with the current research indicates that other studies that reviewed the historical evolution of the complex, the results were obtained only by studying the available written and illustrated sources so the aforementioned studies have not studied the complex of the *Naghsh- e-Jahan square* and the surrounding elements in terms of the geometrical system used in the square and the placement of adjacent buildings. On the other hand, in the articles that have studied the arrangement of elements of the *Naghsh-e-Jahan complex*; the process of development of elements has only been considered from a historical point of view and the effective geometrical factors of the placement of buildings adjacent to the square have not been considered. Also, the studies that are based on the identification of the geometrical principles used in the mentioned complex, are focused solely on evaluating the principles of visual composition of the form and the quality of application of the geometrical systems in the *Naghsh-e-Jahan square* and other related buildings have not been considered. Therefore, on the one hand, the difference between the present study and previous similar studies, is the reason for innovation in this article, The measurement of the geometrical principles used in the *Naghsh-e-Jahan square* was obtained using the relative square measurement system; on the other hand, investigating the effect of the mentioned geometrical system on the form of the plan of the mentioned complex and the location of the elements and entrances of the *Naghsh-e-Jahan square* was another factor which has created a difference between this research and other similar studies.

3. RESEARCH METHOD

The present historical research has been carried out for introducing the hidden geometric ordering principles of the plan of the *Naghsh-e-Jahan square*. Also, in addition to historical research methods, descriptive and exploratory ones have also been used.

The first part of the research data has been obtained before the geometric analysis is done by the descriptive method using the library method, to achieve the historical information on the *Naghsh-e-Jahan square*, especially for determining the formation and its original space. This part of the research has been performed by relying on historical books and ancient texts. In this mentioned part of the research, the researchers sought to find the exact and initial location of the *Naghsh-e-Jahan square*. Also, understanding the evolution process of the mentioned square, from its beginning to the Safavid period, has been another topic in this part of the study.

It should be noted that the descriptive research method can be one of the best and most accurate methods, that help of which it is possible to describe the exact characteristics and functions of this complex; Therefore, the descriptive research method was used in this article in the first part of the research for completing the historical research method that used to find the initial location and the formation process of the *Naqsh-e Jahan square complex*.

It is necessary to mention that It is necessary to carry out this part of the research because determining the exact and initial location of the

[1] It is the tomb of Sheikh Safi-ad-din Ardabili located in Ardabil, Iran.

Naghsh-e-Jahan square has a very special role in finding the initial location of the geometrical system to determine the organizing principles used in the *Naqsh-e Jahan square complex*. In such a way, by getting the correct placement of the geometric system, the possibility of making a mistake in recognizing the ordering principles of the *Naghsh-e-Jahan complex* can be minimized.

The other part of the study is based on the geometrical analysis using known geometrical measuring instruments and systems. The exploratory research method is used in the second part of the study.

This section of the research includes the practical method of measurement of the relative system (congruent rotated squares) as one of the organizations that regulate the structural relations in buildings and historical spaces. In this part, an attempt has been made to the geometric ordering system should be had placed in the correct position (*Qaysarieh's Entrance*) in the plan of the complex; with its help, the geometric proportions used in the *Nosh-e Jahan complex* can be determined. In addition, the researchers have sought to find the hidden geometrical rules, with the help of which, the location of numerous entrances to the square can be determined. So in this part, according to the purpose of the research -finding the hidden geometrical principles used in the plan of the *Naqsh-e Jahan square* and the role of the mentioned principles in locating the numerous entrances to the square- an attempt was made to discover these rules by used the relative square measuring system which is one of the well-known geometrical systems for regulating relationships between components in a complex.

Generally, this study has followed a trend that has been based on previous studies on the governing geometric structure of architectural and urban spaces, especially in the *Naqsh-e Jahan square complex*, and developed it.

4. THE *NAGHSH-E-JAHAN SQUARE*, FORMATION, AND STRUCTURE

Understanding the formation process and the creation of the *Naghsh-e-Jahan square* as one of the most important focuses in the magnificent plans of *Shah Abbas* for the physical development of *Isfahan* is very important. Therefore, the gradual construction of this square and its direct effect on the geometric system and its analysis, as well as the historical development of the square formation have been briefly studied initially.

Based on historical documents, the area in which the *Naghsh-e-Jahan square* was constructed in the *Seljuk period* was part of the vast gardens of the southern suburbs of the city of *Isfahan* called *"Naghsh-e-Jahan"* (Al-Esfahani, 1989, p. 33; Honarfar, 1971, p. 395). In the sources, the *Naghsh-e-Jahan garden* is described as an area with many trees and a large mansion that was used until the period time of *Shah-Abbas II*[1]. He destroyed the garden in 1000 BC and turned it into a square (Monnajem, 1987, p. 113). Various, sometimes contradictory, stories have been told in the sources about the square formation. Among the others, in the *"Montakhab-e-Mozafari"*[2] *is hypothesized* the construction of the square in the cemetery of the *Naghsh-e-Jahan* by leveling the tombs of the sultans (Rafiee Mehrabadi, 1973, p. 20). This area was locally known as *Darolhokoomeh*[3] *of Naghsh-e-Jahan* during the period of *Shah Ismaeil* (Al-Esfahani, 1989, p. 33) and, in some ancient sources, the presence of the square and four Bazaars of *Naghsh-e-Jahan* is reported, before the order to starting the construction by *Shah Abbas II* (Shamloo, 1992, p. 277). In addition to emphasizing this point in other sources, the ceremonies were repeatedly held in *Dolatkhaneh*[4] and *Naghsh-e-Jahan* before by the sultan of *Isfahan*, *Farhad Khan*[5] (Aghushte-ei Natanzi, 1994; Turkman, 1971, p. 427 & 529). Therefore, it can be concluded that the design of a rectangular square with four bazaars, one on each side, including *Dokkan-ha*[6] and *Bazzaz-khaneh-ha*[7], caravans and *garmabeh-ha*[8], mosques, and schools, is about three hundred *Jeribs*[9] in the province of *Isfahan* had been before *Shah Abbas* (Janabadi, 1999, p. 759).

According to historical documents, the construction of the *Qaysarieh*[10] and the leveling of the square was one of the first steps that have been taken to build the *Naghsh-e-Jahan* (Monnajem, 1987, p. 112). Another part of the *Naghsh-e-Jahan*, which was considered at the first stages of the

[1] Shah Abbas II, was the seventh Safavid king (shah) of Iran, ruling from 1642 to 1666 A.D.
[2] A historical book by Sadiqah al-Mu'malk, Abrahym bn Asdallah, 1257-1327 AH.
[3] A name for the place of rule and residence of kings.
[4] Place of a government.
[5] Farhād Khān Qarāmānlu (died 1598), was a Turkoman military officer from the Qaramanlu family, to serve as commander-in-chief of the Safavid Empire.
[6] It means shops.
[7] Merceries
[8] Bathes
[9] Acreage, The area of a piece of land measured in acres.
[10] The Qeysarie (gate) is a historical gate at the main entrance of the Bazaar of Isfahan and Qeysarie Bazaar in Isfahan.

planning of the square, is the *"Aali-ghapou"*[1] mansion. Considering the numerous references to the *"Dolatkhaneh Naghsh-e-Jahan"* in historical texts, the possibility of a building in this place is considerable. Also, the study of the construction of the *Aali-ghapou* building in several stages, suggests the establishment of a building to determine the entrance threshold of *Dolatkhaneh* on the west side of the square, which later became the *Ali-Qapu* building. According to the location of the building close to the *Dolatkhaneh* (the first building) perpendicular to the *Qaysarieh*, indicating the width of the square, the distance between the central axis of the *Qaysarieh* and the *Dolatkhaneh* (western facade of the square) is equal to half of the square width of the square. Given the completion time of *Qaysarieh* in 1005 AH and then the *Sheikh Lotfollah mosque*[2] in 1010 AH, the *Qaysariyah* can be considered as the base point for the *Naghsh-e-Jahan square*. On the other hand, it emphasized the existence of a comprehensive plan for the *Naghsh-e-Jahan square* before its design (Pournadri, 2010, p. 28).

5. ACCESS POINTS: THE SQUARE ENTRANCES

Due to the large width of the *Naghsh-e-Jahan square* and its connection with the surrounding urban elements, the access points of the square and their locations are very important. The relationship between the square and the bazaar as the connecting axis of the old city and the newly developed city of *Safavid* resulted in the definition of the main entrance to the square from the north and the axis of *Qaysarieh* (hallway). Accordingly, in addition to the role played as a basic point in shaping the structure of the square, this part has been also known as the most important entrance to the square from the old city and bazaar.

The defined entrances from the northern, eastern, and southern fronts connected the square to residential regions and public urban services. For example, the connection point of the bazaar and the square from the southeastern corner of the square from the *Hasanabad bazaar*[3]. Unlike the definition of the entrance index of *Qaysarieh*, this section has not a definite effect on the square body, the plan, and the façade. In addition, access has also been made from the eastern neighborhoods to the square. The connection has been made through the eastern bazaar without any specific distinction on the square due to the local operation of these accesses. The connection of the southern neighborhoods was mainly through the *Abbasi mosque*[4] considering the mosque's function.

The gates and entrances of the western body of the square were defined as government entrances that established the connection between the square and *Dolatkhaneh*, palaces, and government gardens. The most prominent of these connections was the hallway that passed through the gate located at the *Ali-Qapu Palace* and was especially for the passage of the king and his companions. In addition, the passage of *Posht-e Matbakh*[5], *Darvazh Khorshid*[6] (access to the king harem), and the communication path with the *Chahar Howz square* [7]as one of the two main squares of the *Safavid's Dolatkhaneh*, as a forecourt for the government buildings with a tall and amazing hallway (Rafiee Mehrabadi, 1973, p. 91 & 107) (Rafiee Mehrabadi, 1973, p. 91 & 107), were certain accesses to the *Naghsh-e-Jahan square* from the western part of the square (Shahabi Nejad & Aminzadeh, 2012, pp. 28-30) (Fig. 1).

[1] Ali Qapu Palace is an imperial palace in Isfahan. It is located on the western side of the Naqsh-e Jahan Square and had been originally designed as a vast portal entrance to the Safavid imperial palaces and gardens which stretched from the Naqsh-e Jahan Square to the Chahar Baq Boulevard.

[2] It was built during the Safavid Empire, standing on the eastern side of Naqsh-e Jahan Square, Isfahan (1603 - 1619).

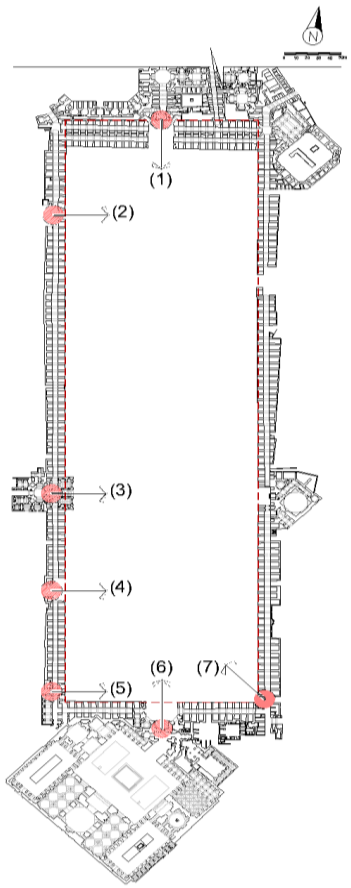
[3] Hassanabad Bazaar is located in the southeast of Naghsh-e Jahan Square and dates back to before the establishment of the Safavid era.

[4] It is a mosque located in Isfahan that is located on the south side of Naghsh-e Jahan Square. It was built during the Safavid dynasty under the order of Shah Abbas I.

[5] One of the gates on the west side of Naghsh-e Jahan Square.

[6] One of the gates on the west side of Naghsh-e Jahan Square.

[7] The square was in front of the Teymouri mansion in Isfahan. This small square had 4 gates, one of which was connected to Naghsh-e Jahan Square.



1. Qaysarieh's Entrance
2. Chahar houze square's Entrance
3. Ali-Qapu's Entrance
4. King Harem's Entrance
5. Posht-e Matbakh's Entrance
6. Abbasi Jammeh mosque's Entrance
7. Hasanabad bazaar's Entrance

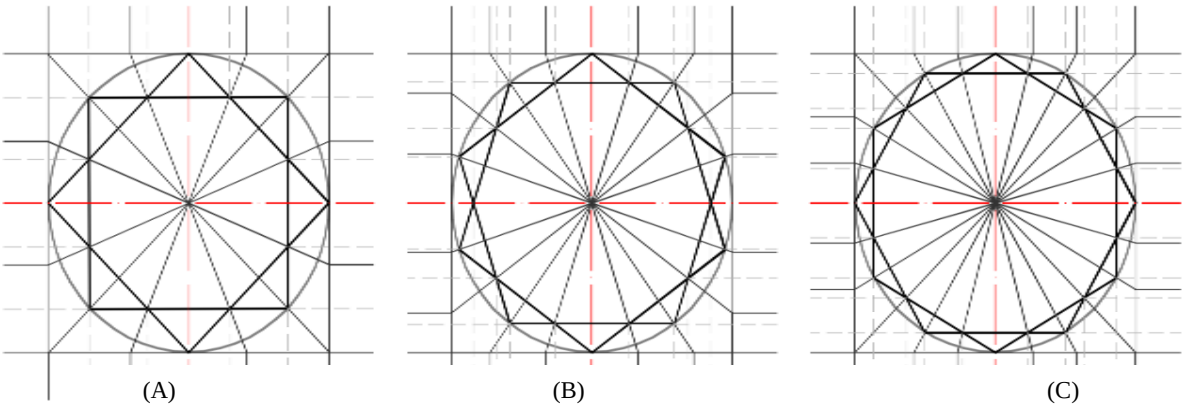
Fig 1. Location of entrances to the *Naghsh-e Jahan square* in the *Safavid era* (writers based on (Shahabi Nejad & Aminzadeh, 2012, p. 139)

6. Geometry Effect On The Formation Of The *Naghsh-E-Jahan Square* And The Locating Of Its Entrances

Considering the hierarchy of urban spaces and their organization, and the ideal combination of mass and space in order to increase the readability of the city's space requires the use of rules that affect the spatial arrangement and the location of the elements of each system. One of these rules is the use of an intelligible geometric system to create identity in urban space. Achieving a certain shape structure in the architectural and urban space requires guidance drawings through which the points, lines, surfaces, and steps to achieve the coordinates of the form are determined. Utilizing a circular base or *parhoon*[1] is one of the most widely used methods to create guidelines for achieving accurate proportions in spaces. The system was based on the division of the base circle into equal parts such as 4, 5, and 6, known as a square, pentagon, and hexagonal system. In this process, a certain relative system was obtained by drawing a surrounded shape within the base circle (the rotation and its internal repeat with a magnitude of 45 degrees). Hence, the guidelines were obtained by the connection between the center of the circle to collision points of surrounding shapes with each other and with the circle[2] (Ranjbar, 2003, p. 51) (Fig. 2).

[1] circle

[2] In this method, and in the first stage, the geometric center of the building is determined, which is in the middle of the yard or the central space of the building. Then, a circle with the center specified in the previous step is drawn. The diameter of the circle, known as the base circle, is equal to the width of the middle space of the building. In the next step, one of the base shapes (square, regular pentagon or regular hexagon) is drawn in the base circle. At the center of the base circle, the mentioned basic shape is redrawn, but this shape is rotated 45 degrees compared to the previous shape. From the center of the base circle to the vertices of the peripheral shapes, as well as the intersection of the above shapes, lines called "guidelines" are drawn, which extend to the peripheral square of the base circle. From the intersection of the guidelines and the mentioned perimeter square, a vertical line is drawn and these vertical lines can determine the spatial division in the building plan.



(A) (B) (C)

A- Relative square Measuring System
B- Relative pentagonal Measuring System
C- Relative hexagonal Measuring System

Fig 2. Devices of Relative Measuring System (Writers)

Therefore, the geometric analysis of the square is considered in this research based on the principles of graphic geometry and the inclusion of a relative system based on $2\pi/4$ that is obtained by dividing the base circle into four parts. The use of this system requires a generating unit to determine the measurement ratio, which is usually considered based on one of the major parts with a given dimension. The base circle is located in the center of the geometry based on this length. In addition, the sides and the diameters of the surrounded square in the base circle, and the circle sectors are considered measuring tools due to the nature of the relative system of geometric measurements (congruent rotated squares). Therefore, the coefficients include 1.2,$\sqrt{2}$ and 2 are considered base ratios. Obviously, the choice of the relative system of square measurement (congruent rotated squares) on the one hand is related to the well-known and widely used division of the *parhoon*, and on the other hand, is performed based on the static geometry used in the ancient Iranian building as a costume in their geometric structure (Moulavi, 2002).

The first stage of geometric measurements of the *Naghsh-e-Jahan square* complex is performed by placing the square relative system on the midpoint of the entrance gate of *Qaysarieh* towards the square[1], in line with the bazaar axis and with regard to the specific width of the square, based on the location of the entrance of *Dolatkhaneh* and the central axis of the *Qaysarieh bazaar*. What is clear in this section is the compliance with the opening of the entrance of the *Qaysarieh* (hallway) at the front of the *Abbasi mosque* as the only entrance to the southern body of the square, from the governing system on the one hand and determining the access to the *Chahar Houz Square* by the middle axis of the system on the other hand (Fig. 3).

In the next step, taking into account the center of the geometric device (point A) and the radial direction along the line closest to the lateral axis of the coordinate to its tangent line (line A-B), an arc is drawn. The meeting point of the arc with the square body at (C) indicates the northern side of the gate entrance of the *Ali-Qapu building* and indicates the entrance to the *Sheikh Lotf al-Allah mosque* along with the mentioned line. In addition, if the arc is drawn with the center of the point of the collision along the previous radius line in the direction of the northern body of the square (point D), the entrance of the *Khorshid* (the king harem) is specified (point E) with an appropriate approximation (the difference is about an opening). Also, by placing the center on the point (F), which is obtained from the collision of the symmetrical radius of the line (A-B) with the tangent line on the system, an arc with the radius (line F-B) is drawn. This line indicates the location of the access to the *Posht-e-matbakh* by the collision with the western body of the square (point G). On the other hand, along the arc by extending the arc as a circle quarter is appeared a point that's named (H); then drawing a horizontal line from it, the southern body of the *Naghsh-e-Jahan* is detected (Fig. 4). It should be noted that since the size of *Naghsh-e-Jahan square* was slightly larger than the current one in the early *Safavid period* (Pirnia, 2010, p. 119), the marginal error of the geometrical body location of the square body within the present range can be ignored.

[1] The *Qaysarieh bazaar* is the main section of the bazaar route, which is known as the main entrance gate (in the northern facade) from the old core of the city to the *Naghsh-e Jahan Square* (Ardalan & Bakhtiar, 2002).

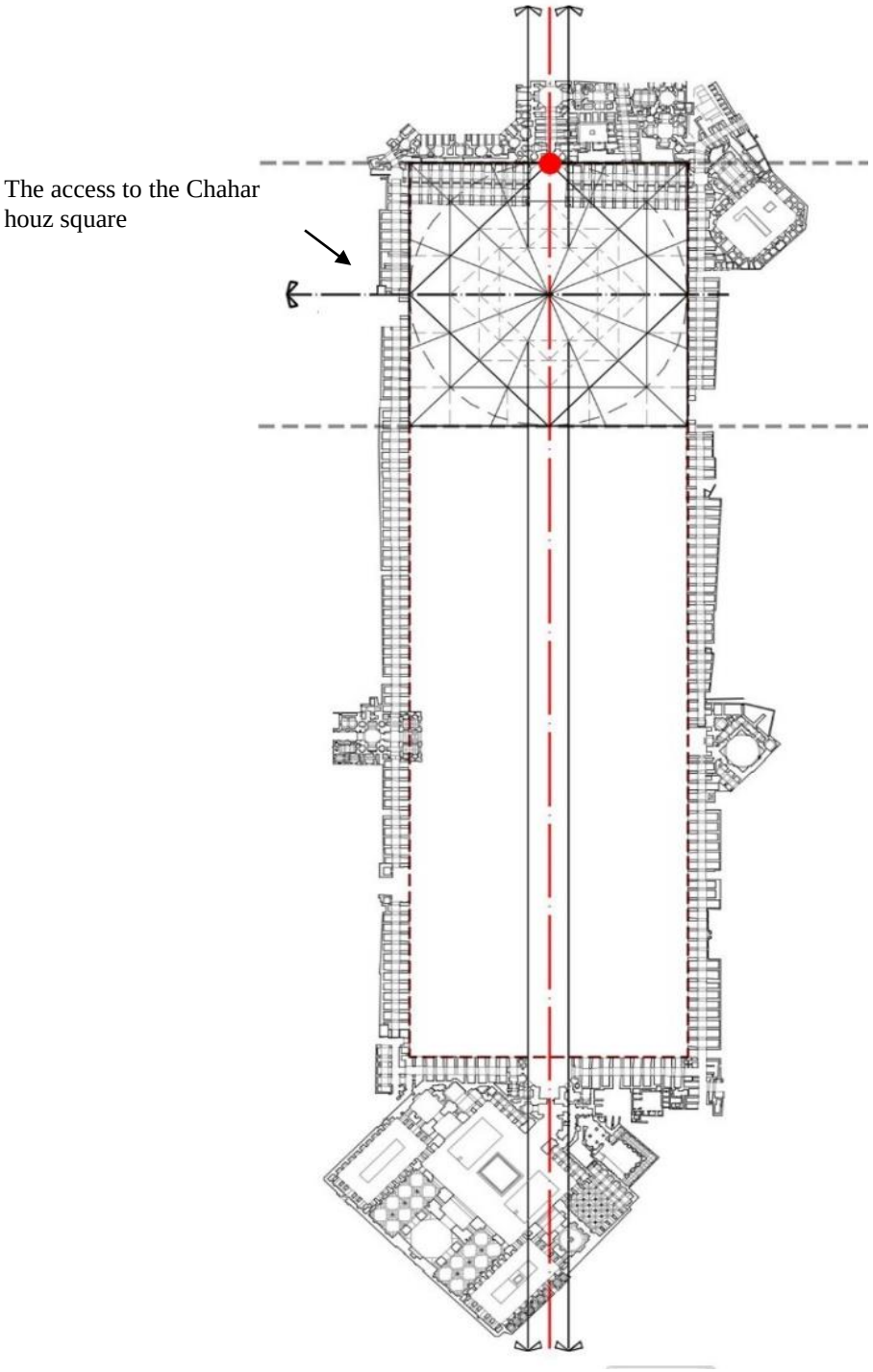


Fig 3. Establishment of the Relative square Measuring system (congruent rotated squares) on the base point of the *Naghsh-e-Jahan square* (entrance of the *Qaisiriyah*) (writers)

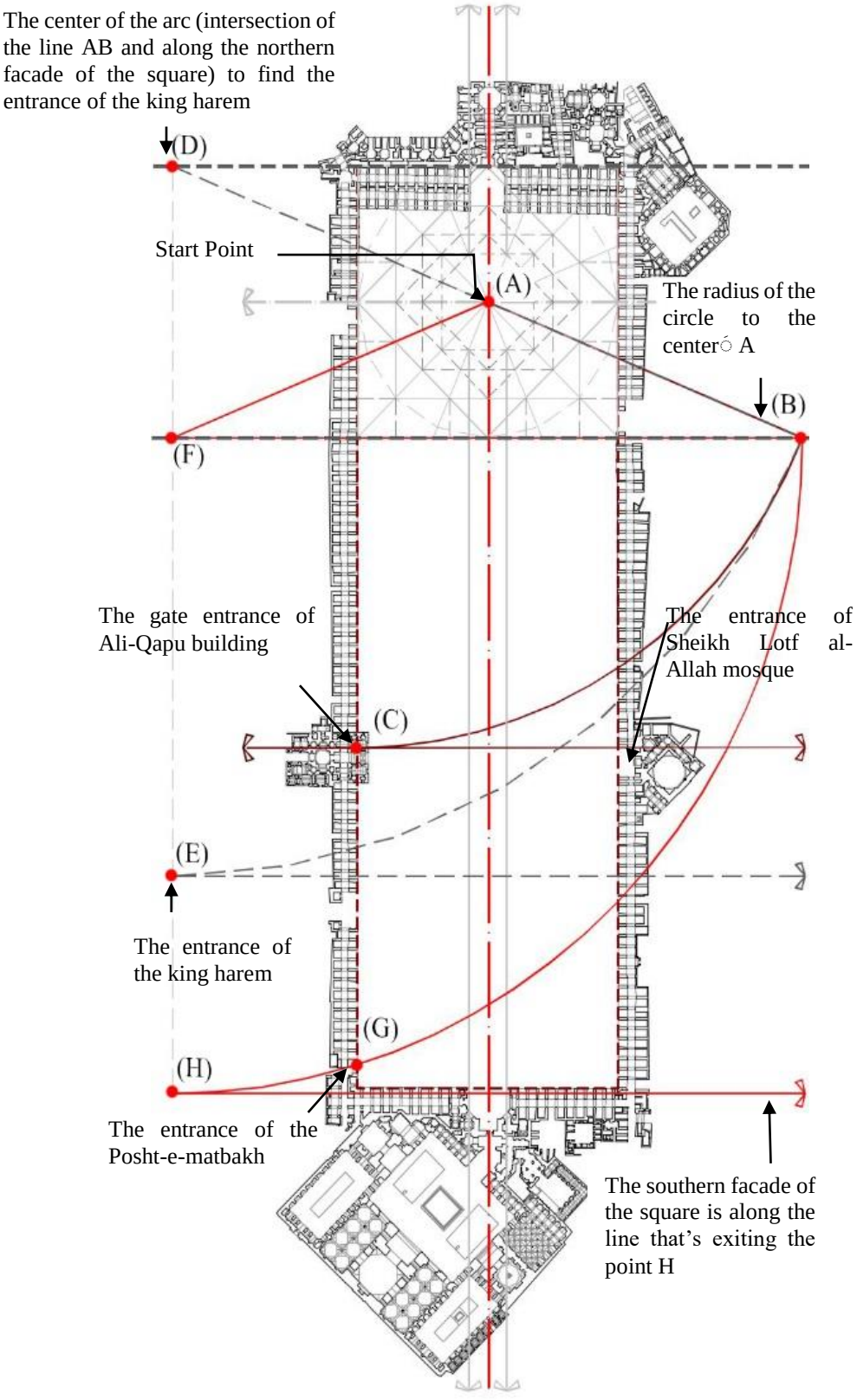


Fig 4. Locating the inputs of the *Naghsh-e Jahan square* based on the Relative square Measuring System (congruent rotated squares) (writers)

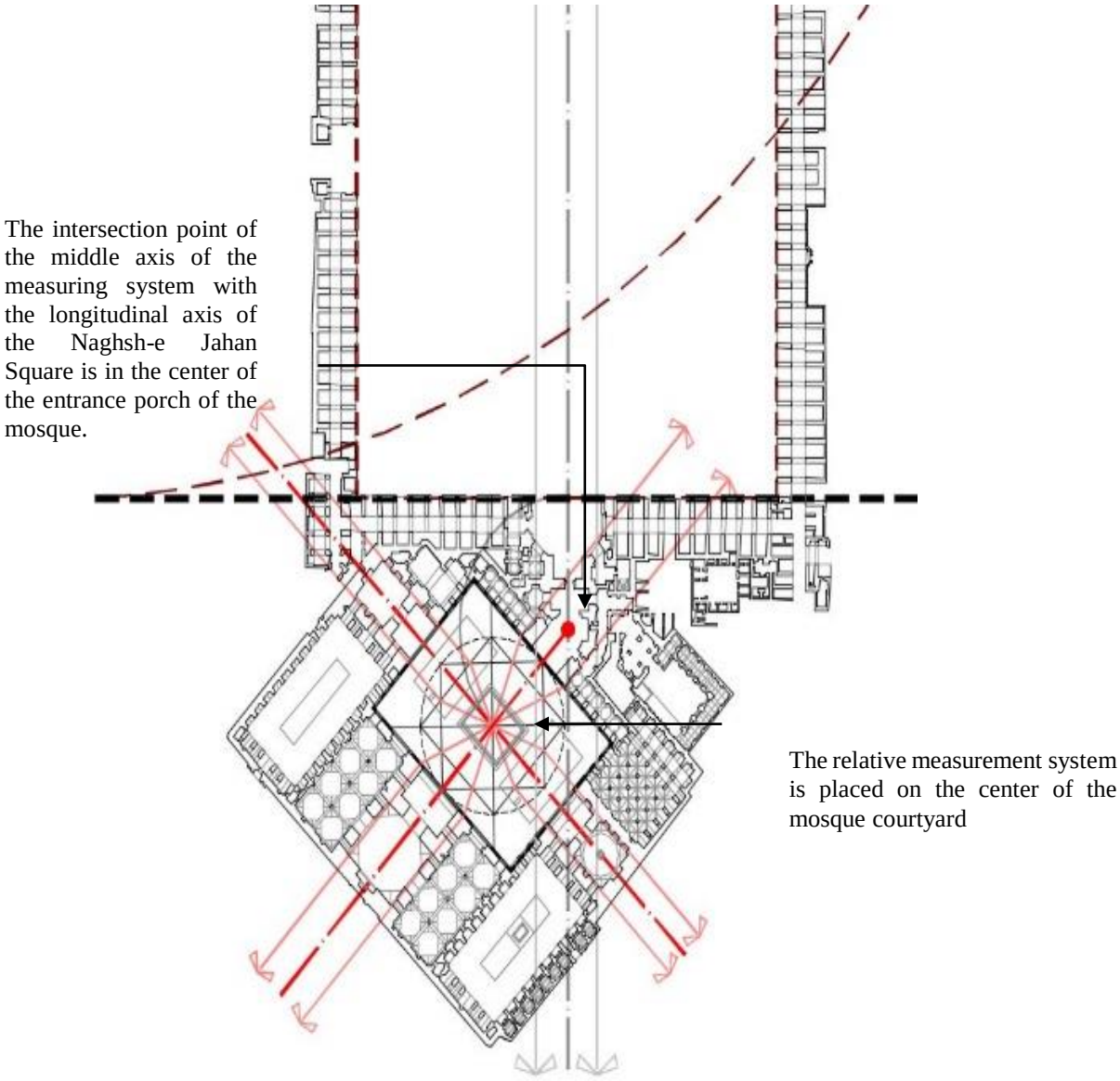


Fig 5. Positioning the Relative square Measuring System on the geometric center of the of *Abbasi Jamee mosque* (writers)

In the following section, to control the effect of the system on the elements around the square, the value of compliance with the relative system of measurement (congruent rotated squares) on the *Abbasi mosque* as the main and most important element of the *Naghsh-e-Jahan square* was considered. Hence, the consistency of this building with the system was evaluated using the establishment of the relative system of measurement (congruent rotated squares) on the geometric center of the mosque. Also, the intersection point of the cross-sectional axis and the longitudinal axis of the *Naghsh-e-Jahan square* were determined (Fig. 5). In the next step, the accuracy of the geometric structure of the square with the governing system on the adjacent elements was examined. At this level, the arc center was placed at point (A) which is the intersection of the axis of the relative system of measurement (congruent rotated squares) with the longitudinal axis of the *Naghsh-e-Jahan square*. The radius was determined as far as the distance from the mentioned point to the geometric center of the *Abbasi mosque*. In this section, the southern body of the *Naghsh-e-Jahan square* was marked (Fig. 6). It should be noted that it is possible to determine the southern crust of the mosque courtyard by placing a square system at the entrance of the *Abbasi mosque*. When the system was located on the openings of the Mosque's entrance by selecting the center of the arc as the point (A) and the radius (A-B), can be achieved that mentioned body (Fig. 7).

7. OUTLINES

The creation of urban spaces based on a complete geometric order expresses pre-designed relationships in the combination of urban elements. This geometrical system has emerged in the structure of the building/city from the morphological point of view. In addition, it can be mentioned the use of geometrical rules by evaluating the relations in the spatial composition between the architectural components, and between the same buildings and the surrounding urban spaces. As per Islamic philosophy, these buildings have been created for showing logical criteria and principles like unity in diversity[1].

It is necessary to mention that the origin of these rules was based on the circle that has the symbolic conception of unity in Islamic philosophy. Hence, the system of the relative measurement based on the number four or square, which consists of dividing the base circle into four, and then eight equal parts, is considered in the present study as a structure that can be used for analyzing the proportion between the set's elements. The analysis carried out on the plan of the *Naghsh-e-Jahan Square* and the *Abbasi Mosque* represents the direct use of relations based on rational proportions and the symmetry hypothesis. Based on geometric drawings used in the recent system, the location of the entrances to the square can be evidence of the advanced design of this great space. Also, following the spatial division of the *Abbasi Mosque* from the square (entrance gate, the four openings of the porch, the dome, the side of the courtyard of the secondary school, etc.) and the continuous structure of its components based on the system that is located in the square (geometric center location of the courtyard and its southern end), exhibits an idea that aims to unify the various parts of the city within the largest architectural element in the *Naghsh-e-Jahan Square*.

8. CONCLUSIONS

Understanding how the coordinated geometric system works in urban spaces and architectural buildings is essential from two perspectives. On the one hand, the organization of the space is a philosophical thought that represents a degree of unity of existence in the universe; on the other hand, it introduces the governing geometric relations of the structure based on the principles and rules of the Euclidean order. This method is based on the fundamental principles that achieve unity through repetition and elaboration by choosing a single form.

The reading of a hidden geometric system was considered in the present study and was applied to the *Naghsh-e-Jahan square* and the spatial relationships between its elements based on Euclidean geometry rules. In this regard, the obeying of this part from the recent structure became clear by establishing the relative fit of the four at the beginning point of the structure of the *Naghsh-e-Jahan square* and according to its predetermined width based on the location of the *Safavid Dolatkhaneh* (*Ali-Qapu*).

In addition, the assessment of the construction process of the *Naghsh-e-Jahan square* emphasizes the special attention to design for locating the access points of the square. In this process, it was important to put the base point of the structure of the square in communication and connection with the old city and the new structure (bazaar) and to consider the specific impact of the connection on the size and, consequently, on the governing proportions. Adherence to the system of access to the square (main entrances) from the system located at the entrance of the *Qaysarieh* and the adherence of the peripheral elements, especially the *Abbasi mosque* to this structure was also identified in this analysis. Therefore, according to various aspects and layers that in relation to each other organize the semantic combination of the city/building and the method of locating urban and architectural elements, the geometric dominance of the *Naghsh-e-Jahan square* can be considered as the translation of the concept of a recognizable space based on geometric logic in the design of the modern *Safavid city*.

[1] For more information, refer to: (Ardalan & Bakhtiar, 2002).

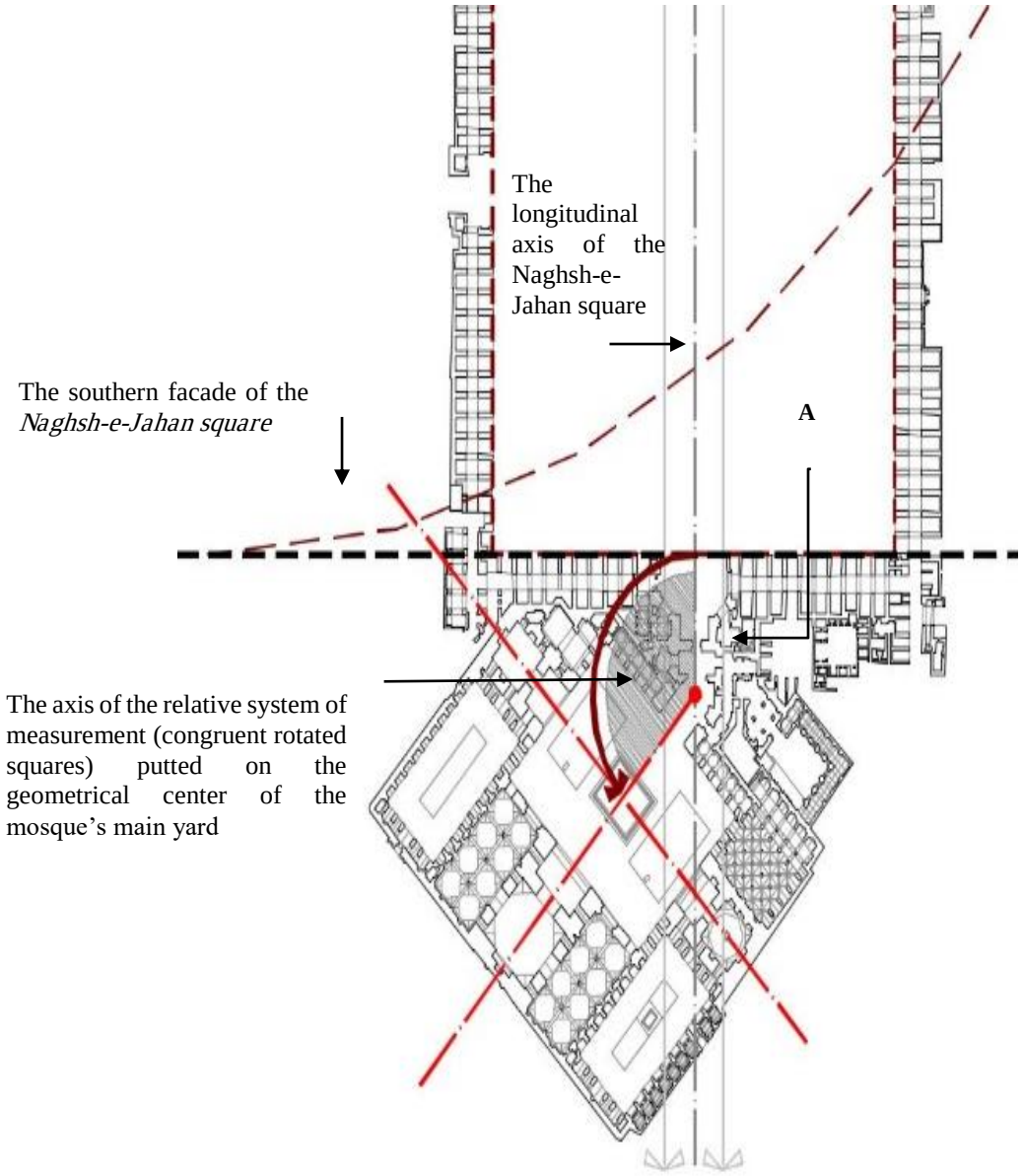


Fig 6. Determining the southern body of the square with the establishment of the relative measurement system of the square (congruent rotated squares) on the geometric center of the *Abbasi Jamee mosque* courtyard (writers)

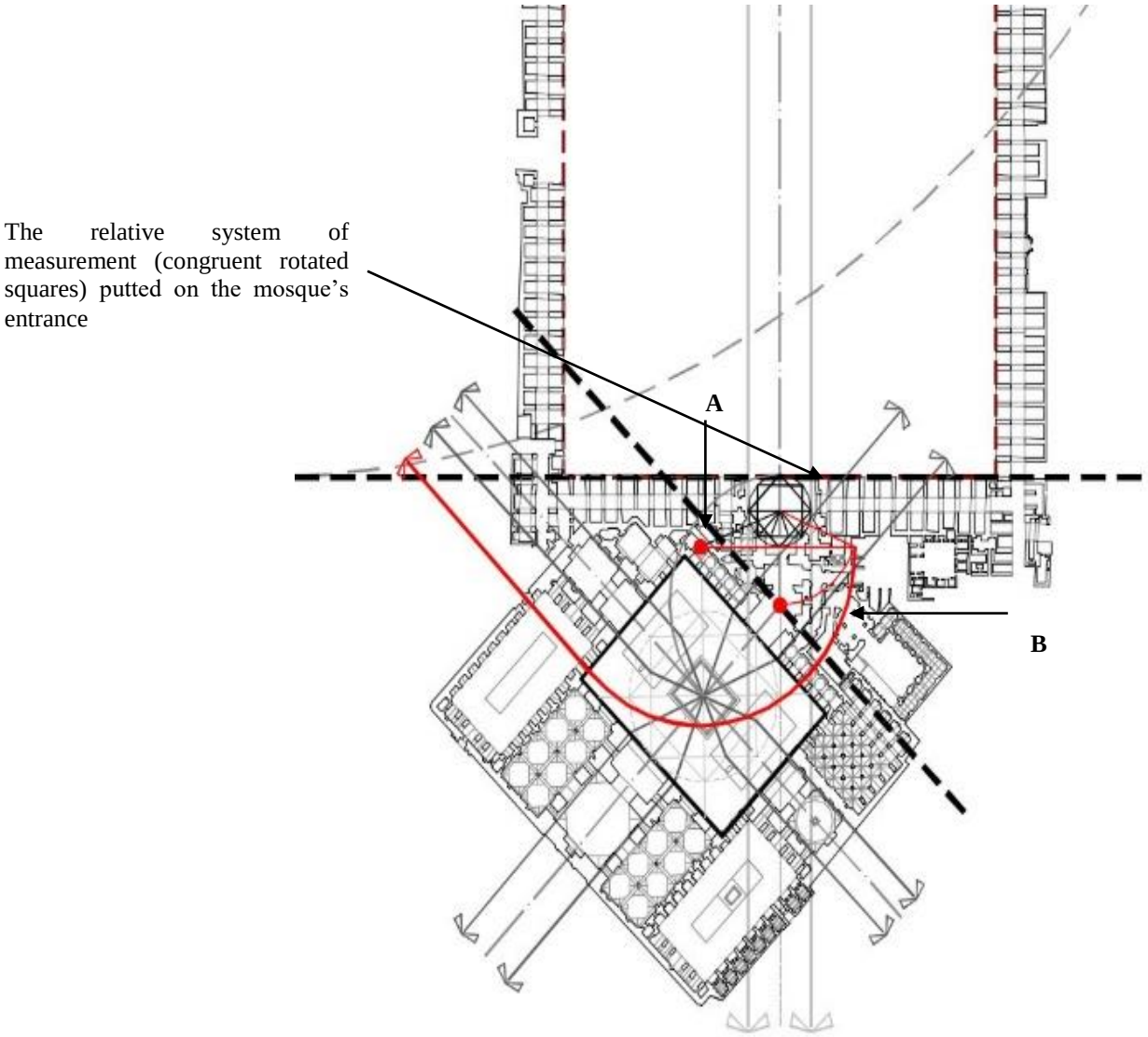


Fig 7. Determination of the width of the courtyard of the *Abbasi Jamee mosque* based on the Relative square Measuring System (congruent rotated squares) on the entrance of the building (writers) g

REFERENCES

Aghabozorg, N., & Motadayyen, h. (2015). The theoretical origin of the Naghsh-e-Jahan square. Bagh Nazar, 12(33), 23-40. Retrieved from http://www.bagh-sj.com/article_9982_f5c2a345a567b12ecc4a60a00e04da34.pdf

Aghushte-ei Natanzi, M. (1994). Faces and Figures The Cultural Heritage of Felicity in Safavid History (A. Eshraghi Ed.). Tehran: Scientific and Cultural Company.

Ahari, Z. (2006). Isfahan School in Urbanization: Urban Designer Grammar. Tehran: Ministry of Culture and Islamic Guidance, Printing and Publishing Organization; Arts Academy Publishing.

Ahmadi, F., Asadpour, A., Pouryoosef Zadeh Najafabadi, S., & Azari, R. (2011). Restoration strategies of naghsh-e-jahan square to achieve social sustainability based on users visions. Elixir Soc. Sci, 39, 4985-4992.

Al-Esfahani, M. (1989). Half of the world in the definition of al-Isfahan (M. Sotoudeh Ed.). Tehran: Amir Kabir.

Al-Saeed, A., & Parman, A. (2010). Geometric patterns in Islamic art (M. Rajabnia, Trans.). Tehran: Soroush.

Ardalan, N., & Bakhtiar, L. (2002). Sense of Unity (H. Shahrokh, Trans.). Isfahan: Khak.

Dahar, A., & Alipour, R. (2013). Geometrical analysis of the architecture of Sheik Lotfollah Mosque to find the geometrical relations between its prayer hall and the entrance. Bagh Nazar, 10(26), 33-40. Retrieved from http://www.bagh-sj.com/article_3446_5bb8206261fe4ae88eb7be9c660ff164.pdf

Dizani, E. (2014). Explaining the evolution of Shah square pattern in Safavid capitals of Iran. Hoviatshahr, 8(17), 77-86. Retrieved from https://hoviatshahr.srbiau.ac.ir/

article_3170_845d3b9ae5eb122102160cc3092508be.pdf

Falahat, S. (2018). Cities and Metaphors (Beyond Imaginaries of Islamic Urban Space). London & NewYork: Routledge.

Habibi, M., & Ahari, Z. (2012). Isfahan School: Urban Design Language in Ancient Cities. Tehran: Office of Cultural Research.

Hojat, I., Balali Oskouyi, A., & Yazdani, S. (2019). Comparative study of physical and functional structure in Tabriz Saheb-Abad square and Isfahan Naghsh-e-Jahan square in hictorical periods. Journal of Research in Islamic Architecture, 7(1), 21-42. Retrieved from http://jria.iust.ac.ir/article-1-1128-fa.html

Honarfar, L. (1971). Treasures of Historical Monuments of Isfahan. Tehran: Ziba printing office.

Jafari Jabali, H., & Shahabi Nejad, A. (2016). Historical city squares with a special look at Imam Ali Square (Atiq) of Isfahan. Isfahan: Isfahan Cultural and Recreational Organization.

Janabadi, M. (1999). Rostai al-Safavid (T. Majd Ed.). Tehran: Endowofat Afshar Publications.

Mahboobi, G., Mokhtabad, M., Etesam, I., & Attarabbasi, M. (2018). Symbiosis of the Inside and Outside in Architecture of the Naqsh-e Jahan Square. Bagh Nazar, 15(58), 51-64. Retrieved from http://www.bagh-sj.com/article_59577_726841bc1f41d157df00bf3586fd029a.pdf

Mojtahedzadeh, M. (2013-06-26). Geometry at work: Re-reading the Persian bazaar. Paper presented at the 1st International Conference on Architecture and Urban Design. http://dspace.epoka.edu.al/handle/1/241

Monnajem, M. (1987). Abbasi history or Mullah Jalal newspaper (S. Vahidnia Ed.). Tehran: Vahid.

Moulavi, B. (1991, summer). Geometric drawing method in Iranian architecture (Islamic era). SOFFEH, 1(2), 73-85. Retrieved from https://soffeh.sbu.ac.ir/article_99494_2753a182343a9964575ce7029e73ffa6.pdf

Moulavi, B. (2002). Investigate the Application of Geometry in the past Architecture of Iran (Islamic era). Tehran: Building and Housing Research Center.

Najiboglu, G. (2010). Geometry and Ornamentation in Islamic Architecture: (Topkapi Scroll) (M. Ghiyomi, Trans.). Tehran: Venus.

Pirnia, M. (2010). Familiarity with the Islamic architecture of Iran (urban and suburban buildings) (G. Memarian Ed.). Tehran: Soroush Danesh.

Pournadri, H. (2010). Development and Formation of the Dolatkhaneh of the Naghsh-e-Jahan, Ali-Qapu (A commentary on the studies of the Ismeo's Institute for the Historical Courseology of Ali Qapu). SOFFEH, 20(51), 23-30. Retrieved from https://old.sid.ir/fa/journal/ViewPaper.aspx?ID=131439

Radhamadi, M., Tehrani, F., & Aboei, R. (2011, summer). Introducing and criticizing several recent historical image documents on the Naghsh-e-Jahan square. Bagh Nazar, 8(17), 3-18. Retrieved from http://www.bagh-sj.com/article_112_49d016b41499709a07b2214dacfc4351.pdf

Rafiee Mehrabadi, A. (1973). Isfahan National Remnant. Tehran: National Library of Iran.

Rahbar, M., & Ansari, M. (2016). Isfahan's Organic and Planned Form of Urban Greenways in Safavid Period. International Journal of Architecture and Urban Development, 6(1), 21-32. Retrieved from https://ijaud.srbiau.ac.ir/article_9500_4c8068abb082b6af0f0cda15168f561d.pdf

Rahmani, M., Nasiri Kia, S., Kazemzade Raef, M., & Mir Derikondi, S. (2022). Feasibility of influencing the urban spaces of the European Renaissance period in Naqsh Jahan Square. Shabak Scientific-Specialist Journal, 6(7), 157-168.

Ranjbar, M. (2003). Introduction to Iranian Measure. Tehran: Iranian Mehrz Society.

Shahabi Nejad, A., Abuei, R., & Ghaleh Noei, M. (2016, Februry). The open space of the Naghsh-e-Jahan square; the Values and issues. Bagh Nazar, 13(44), 53-64. Retrieved from http://www.bagh-sj.com/article_42145_18444959a2e611d37b38c450006a6009.pdf

Shahabi Nejad, A., Abuei, R., Ghaleh Noei, M., & Emami, S. (2014, spring and summer). Historical Formation and Metamorphosis of the Naghsh-e-Jahan square. Specialty Faculty for Restoration of Cultural Heritage, 2(new period3), 45-65. Retrieved from http://www.aui.ac.ir/index.php/fa/rsch

Shahabi Nejad, A., & Aminzadeh, B. (2012). Paradise entrance to the Naghsh-e-Jahan square of Isfahan: ; Values and issues. Honar hai-e Ziba (Architecture and Urbanism), 17(1). doi:10.22059/jfaup.2012.29694

Shamloo, V. (1992). Ghesas al-Khaqani (S. S. Naseri Ed.). Tehran: Publishing and Printing Organization of the Ministry of Culture and Islamic Guidance.

Turkman, A. (1971). The History of the Abbasid Opinion (A. Afshar Ed. Vol. II). Tehran: Amir Kabir.

Utaberta, N., Mamamni, H., Surat, M., Che-Ani, A. I., & Abdullah, N. A. G. (2012). The Study on the Development of Ornamentation in the Architecture of Safavid Dynasty. International Journal of Architectural and Environmental Engineering, 6(7), 465-469. Retrieved from https://publications.waset.org/1879/the-study-on-the-development-of-ornamentation-in-the-architecture-of-safavid-dynasty

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