

## Research Paper

# Quantitative investigation on shaded area according to the geometry of blue-mosque domes in Tabriz-Iran

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### Abstract

The numerical investigation of computing the rate of radiated solar-energy requires equations as a function of solar-time according to the radiation angle of the sun ( $\beta$ ) in the latitude  $37^{\circ}\text{N}$  and the shape. This paper implements the Lagrange interpolation to obtain the equations of normal diagram of elements based on data obtained from surveying. Also the method of Hann-window equation used to estimate the sun path. Albeit, the solar radiation angle effects on the length of element's shadow on the ground but it is a function of solar-time too. The parameters of the dome traits did compute using the solar-geometry principles in a coexisted function of time to reach a viable equation. The amount of absorbed and wasted energy is presented by analysing its surfaces in the all sides simultaneously. Every single element of BM has been considered to define its especial equation of geometry to analyse their different reactions while transferring energy. Quantitative method of this paper has based on library method of research to define especial equations and field survey to experiment data analysis. According to the mentioned methodology, around  $7,54e+4$  (Kcal.h/m<sup>2</sup>) solar energy radiate on the BM that its value is equal to around 9,43 (lit/h) energy of gasoil. In other word at least 75,44lit/day fossil energy is saved by the Blue-mosque.

**Keywords:** Radiation energy, Domatic forms, Shaded area, Nominal absorption.

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## 1. INTRODUCTION

The Blue-mosque (Göy Məçit in native language) of Tabriz was designed in order of Jahan-Shah who was a Turkish king from tribe of Qara-Qoyunlular in 1448 AC. Its architectural style in Iranian art categorized in Azeri-style [1]. It is the masterwork of Azeri-style especially among the Islamic-styles because it is well known as Turquoise of Islam that reflects its tiles hue. However, there are some traces of Ottoman architecture in its formation.

It should be mentioned that the majority of masterworks in ancient Azerbaijan-Iran obey their climatic accordance by implementing of different architectural techniques to design their building according to own methods.

This paper investigates the amount of solar energy that can be radiated on the Blue-mosque in Dec 21<sup>st</sup>. It is essential to determine the effective parameters on solar-energy to perform calculation. There are two types of parameters.

The locational latitude and formal traits of elements those define the place functions and the general specification

of solar motion on its path that defines the time functions [2], [3], [4].

In other word, this paper subtracted the amount of shaded area where is depend on the elements heights after calculating the rate of solar energy in latitude  $37^{\circ}\text{N}$ .

It is necessary to categorize the type of effective elements like domes in classified groups to define their equations as functions of distance that accelerates the process of calculation the amount of shaded areas [5]. However, the amount of shadow area on the ground depends on the element height and the radiation angle of the sun that is a function of time [6], [7], [8].

The sun downs in the lowest level in the winter solstice in north hemisphere and the length of shadows increase on maximum amount during the year [9], [10].

Because the building of the Blue-Mosque is located in a cold and frozen climate, subsequently, analysing the building traits in this time helps us to judge about its architectural techniques as a viable and functional view according to the climatic requirement [11].

In this paper the amount of shaded area is calculated to find the amount of solar energy radiates on blue-mosque surfaces and how much the Blue-mosque can absorb of that.

The maps and details of the Blue-mosque have been drafted to data mine. The rate of the solar energy has known to its location and time but the rate of the

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absorption energy depends on the surfaces, colour, materials, formation, locating aspect and locating angle of the building. This paper focused on the surface parameters. The maps and details of the B.M. are drafted to determine effective geomatic parameters.

Previous investigations on BM were limited about its climatic analysis or just qualitative and aesthetically analysis. It should not be forgotten that the elements of BM had never analysed according to the solar geometry by mathematic methods that computes accurate amount of energy transferring before this paper. Because every single element of BM has been considered to define its especial equation of geometry to analyse their different reactions while transferring energy.

Defining especial equation of element's geometry made it possible to analysis BM thermal behaviour to compute the rate of absorbed, saved and transferred energy in different aspects simultaneously. This quantitative method is novel and several times accurate than using sun-path diagrams that is usual in sustainable architecture.

Another advantage of this method is its performance to determine the range of elements shadow that could be useful for restoration of neighborhoods.

## 2. SOLAR TIME AND RADIATION ENERGY RATE (T)

The rate of solar radiation energy ( $e$ ) is a function of radiation angle ( $\beta$ )[12],[13]. This angle is a function of time ( $t$ ) in different latitudes. So, both of them can be defined as a function of time. Because the rate of radiated solar energy changes constantly by changing its radiation angle according its latitude, the equation of the average rate of radiated solar energy in latitude  $37^\circ\text{N}$  in winters can be calculated using Lagrange parabolic interpolation as follow;

$$e(t) = \frac{(T - t_1)(T - t_3)}{(t_2 - t_1)(t_2 - t_3)} (e_2) \tag{1}$$

It should be mentioned that the surfaces of north aspects do not considerable according to azimuth in the winters of north hemisphere [14].

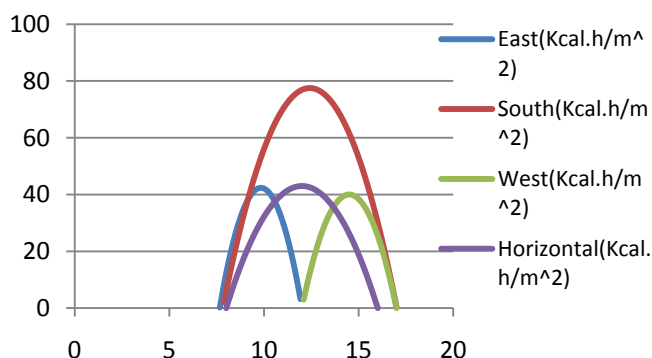


Fig. 1 Diagram of radiation energy on latitude  $37^\circ\text{N}$

The amount of the radiated solar energy can be

calculated by integration as follow: [15]

$$\sum E(t) = \int e(t)dt \tag{2}$$

Aspect	T <sub>(H)</sub>	e <sub>Kcal.h/m2</sub>	E <sub>(Kcal.m2)</sub>
East	07:40-12:00	26	113
South	08:00-17:00	50	449
West	12:00-16:00	31	125
Horizontal	08:00-16:00	27	215

Because the calculated amount of solar energy varies on the every point of latitude  $37^\circ\text{N}$ , the maximum rate of radiated solar energy is considered as the traits of architectural details.

## 3. THE LENGTH OF SHADOW ON THE GROUND (L)

The length of horizontal shadow on the ground is a function of elements height and the angle of radiation [16].

$$l = \frac{h}{\text{tg}\beta} \tag{3}$$

Where: ( $l$ ) is the length, ( $h$ ) is height of the element and  $\beta$  is radiation angle. According to Fig. 2 there are 7 types of domes based on their size. Subsequently, this paper reviews the general relation as a function according to the every detail of the elements separately.

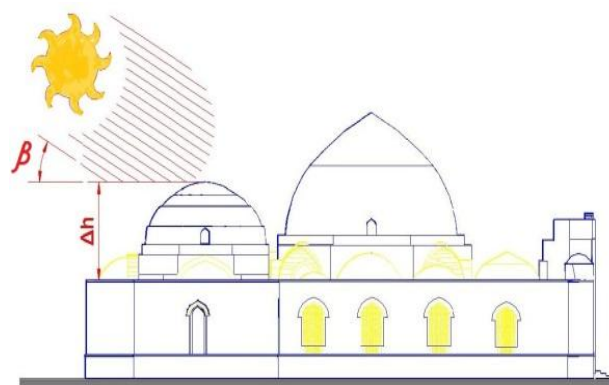


Fig. 2 Radiation angle and element height

### 3.1. Elements equations $h(x)$

As mentioned before, it is essential to describe the schematic shapes of elements as their geometries diagrams by attending to their equations. The size of each element is measured after accurate mapping of Blue-mosque. The form of Blue-Mosque includes two cubes with (8.7<sub>m</sub>) height. Also, there are thirteen domes on its roof. The similar domes are arranged into same category. These domes are categorized as Fig. 3. Obviously, each type of shapes requires definition of its equations to estimate the rate of heights in different points. Free diagrams of the

elements are analysed to define its equations after their model as Fig. 4.

Construction of domes with parabolas and circles in their arches were prevalent in Azerbaijani (Turan-Iran)-architectures. Compounded forms have structural and

aesthetical reasons but their roles are not considerable on shadow rate [16]. On the other hand, the affective distance to length of shadow is the maximum height of element. For example the normal diagram of dome (C4) that implied different shapes is presented as follow;

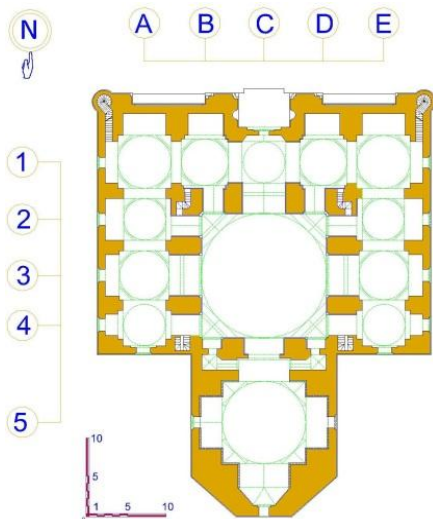


Fig. 3 Plan classification

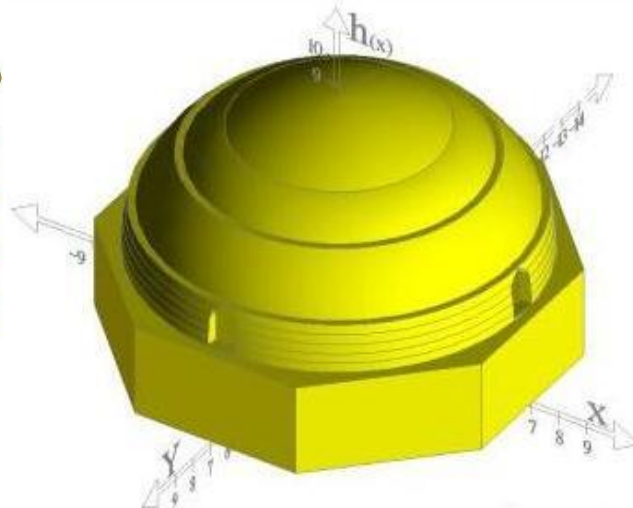


Fig. 4 Model dome class (C5)

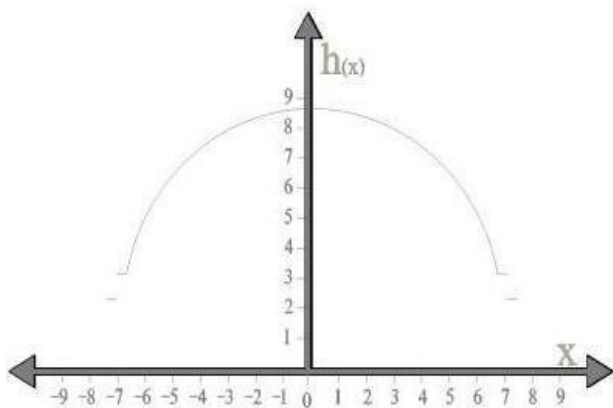


Fig. 5 Normal diagram (C5)

The equation of elements defines its height according to its horizontal distance (x). To condensate, the results of analysed diagrams are presented in Table 2.

The method of Lagrange interpolation is used to calculate the elements equations. These equations are used to calculate the shadow area on its roof. Also the equation of radiation angle describes the relation of area and time.

### 3.2. Radiation angle $\beta(t)$

The rate of radiation angle decrease until  $30^\circ$  in latitude  $37^\circ N$  in winter solstice according to the Blue-mosque location. The equation of radiation angle calculated by the Hann window functions as follow [17];

$$\beta(t) = \frac{77}{5} \left( 1 - \cos\left(\frac{\pi}{6}\right) \right) + \frac{7}{10} \left( 1 - \cos\left(\frac{\pi}{90}\right) \cdot t \cdot \pi \right) \quad (4)$$

Table 2 Elements equations

Group	Equation of Height	$D_h$
$A_1, E_1$	$h(x) = -\frac{11}{50}(x^2 - 7x)$	$[0, +7]$
$A_2, E_2$	$h(x) = -\frac{1}{4}x^2 + \frac{23}{20}x$	$\left[0, +\frac{5}{2}\right]$
$A_3, E_3$	$h(x) = -\frac{3}{25}\left(x^2 - \frac{73}{10}x\right)$	$\left[0, +\frac{36}{5}\right]$
$B_1, D_1$	$h(x) = -\frac{1}{8}x^2 + \frac{31}{40}x$	$\left[0, +\frac{12}{5}\right]$
$A_4, C_1, E_4$	$h(x) = -\frac{27}{100}x^2 + \frac{17}{200}x$	$\left[0, +\frac{17}{5}\right]$
$C_3$	$h(x) = -\frac{1}{6}(x^2 - 19x)$	$[0, +19]$
$C_5$	$h(x) = \frac{9}{2} + \sqrt{47 - x^2}$	$\left[-\frac{137}{20}, +\frac{137}{20}\right]$

## 4. SHADED AREA

The results of the cubic forms of the Blue-mosque are not effective on the shaded area because their shadow do not cover the roof. In other word there is not any bar in at any aspects of them that disturbs the radiation process but the most decisive points are just domical elements. Subsequently, this paper emphasized the role of domes.

As it is mentioned, the length of horizontal shadow (l) of elements on the roof is a function of element height (h) and the radiation angle ( $\beta$ ). Relation of state as a function

of time and element distance presented as follow;

$$A_{(x,t)} = \int \alpha(t,x) dt = \int \frac{h(x)}{\text{tg} \left[ \frac{77}{5} \left( 1 - \cos \left( \frac{\pi}{6} \right) \right) + \frac{7}{10} \left( 1 - \cos \left( \frac{\pi}{90} \right) t \cdot \pi \right) \right]} dt \quad (5)$$

This equation is the integration of the shadow function on the ground (roof plus neighbors) that is a function of sun path and radiation angle. Its nominator is the tangent of radiation angle that is the function of solar time.<sup>5</sup> The rate of shaded area can be calculated just as a function of time in the mentioned equation by applying the numerical amount of element height ( $h(x)$ ) [18],[19],[20].

The amount of shaded area must be subtracted of total roof on its surface. The outcome of the mentioned equation is presented for the element (C4) that is the highest element with maximum overlaps. The path of important element is presented in Fig. 6. Also, the shadows of total elements simulated in three dimensional modelling at 09:00 A.M. in Fig. 7.

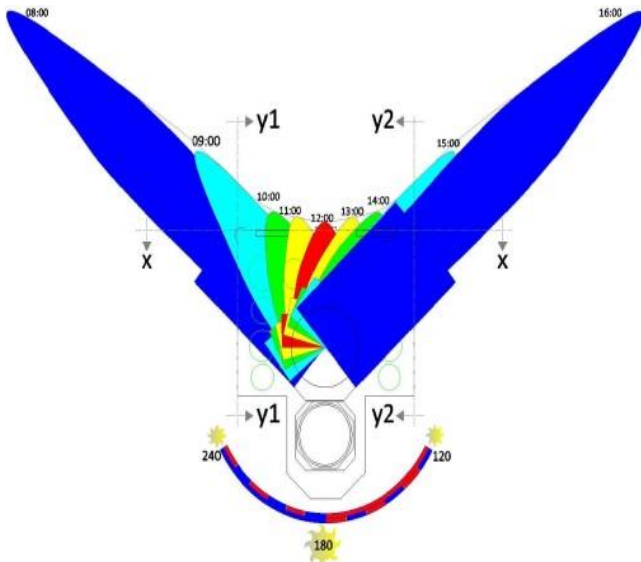


Fig. 6 Shaded area path of C3

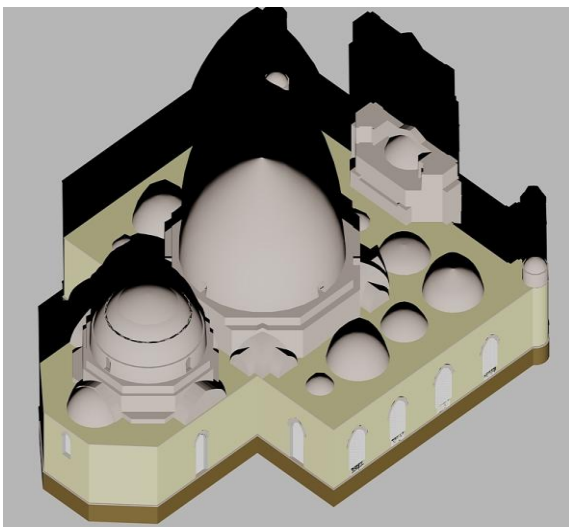


Fig. 7 3D modelling of shaded and exposed area

According to the mentioned calculations, the amount of shadow area of each element had been presented separately but the majority of shadow area is not effective on the roof. The effective shadow area is presented after subtracting the overlapped parts and extra zones during sunrise till sundown in Dec 21<sup>st</sup>. The amount of effective shadow area on the roof is the area between limits (X-X) and (Y<sub>1</sub>-Y<sub>2</sub>). The rate of shadow and exposed area is presented as follow;

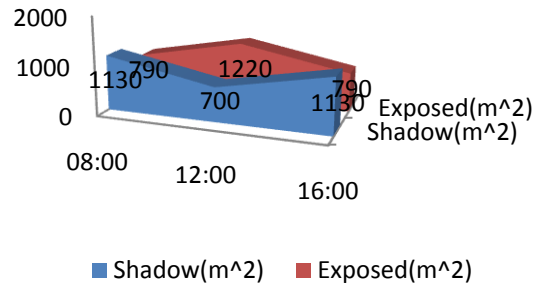


Fig. 8 The rate of shaded and exposed area of the roof

## 5. EXTRA AREA

According to the mentioned data averagely 41% of roof surfaces are under the shadows of the elements that subtracted from total rate.

On the other hand, the other sites of the domical elements expose by the sun that increase the amount of the radiated surfaces.<sup>6</sup> The extra surfaces(s) of the exposed aspects obey of the relation as a function of element volume ( $v$ ) and radius( $r$ ) as follow;

$$S \cong \sum_{i=1}^{n_i} \left( \frac{180 - \left[ \frac{77}{5} \left( 1 - \cos \left( \frac{\pi}{6} \right) \right) + \frac{7}{10} \left( 1 - \cos \left( \frac{\pi}{90} \right) t \cdot \pi \right) \right]}{360} \right) (2\pi r^2) \quad (5)$$

The results of the exposed surfaces those provide extra areas are presented for each element separately as follow;

Table 3 Exposed Area of Every Element	
Type	Extra Surfaces(m <sup>2</sup> .h)
A <sub>1</sub> ,E <sub>1</sub>	2*(34)
A <sub>2</sub> ,E <sub>2</sub>	2*(19)
A <sub>3</sub> ,E <sub>3</sub>	2*(40)
B <sub>1</sub> ,D <sub>1</sub>	2*(15)
A <sub>4</sub> ,C <sub>1</sub> ,E <sub>4</sub>	3*(3)
C <sub>3</sub>	515
C <sub>5</sub>	180
Total	920

The calculations show that, averagely 41% of all surfaces on the roof overlap each other. Subsequently 41% of 1920<sub>m<sup>2</sup></sub> equals around 787<sub>m<sup>2</sup></sub>. The proportion of shaded area, extra surfaces and total area of the roof have presented in Fig. 9.

This figure does illustrate the role of elements on the roof of the Blue-mosque. Its elements just waste 13% of total area on its roof in the worst condition (winter solstice) when the sun stands on the lowest level and the lengths of shadows increase to its maximum level.

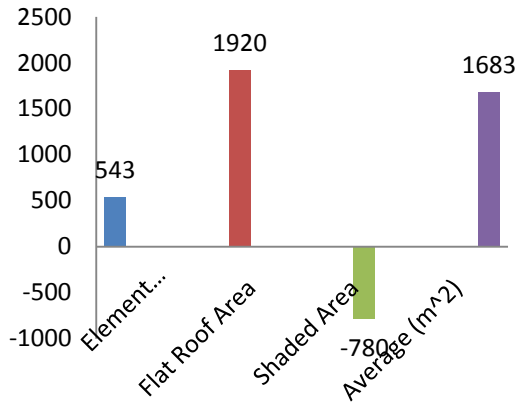


Fig. 9 All the effective items on the roof area

## 6. ABSORPTION

It must be mentioned the purpose of absorption is nominal absorption that is the maximum rate of solar radiation on building that it could be absorbed in an ideal situation without considering any wasted amount.

### 6.1. Absorption ability

The absorption ability is apart from solar-geometry but obeys it. In other word, the amount of Solar-energy radiates on every point in the same latitude equally and the amount of energy absorption depends on several factors such as form and shapes, type and thickness of materials and the pure rate of surface area of roof, the rate of shaded area and the amount of elements surfaces define the absorption ability in this paper according to the radiation amount [21],[22]. The amount of effective factors on the each aspect of Blue-mosque is presented to find the amount of roof as follow;

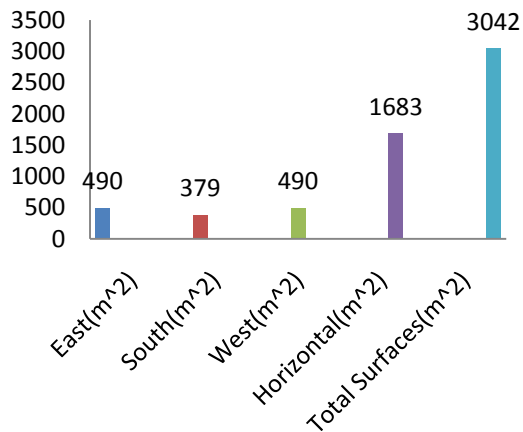


Fig. 10 Surfaces area

### 6.2. Energy value

Analysing all the effective factors of the Blue-mosque elements made it much viable to calculate the amount of surfaces in different aspects. This chapter calculates the value of solar energy according to the solar geometry and building traits. The second chapter of this paper considered the condition of solar energy radiation in the winter for known latitude and the prior part purposed the amount of viable surfaces in different aspects. All of these data are essential to estimate the nominal energy. The average amount of solar energy that radiates on different surfaces of Blue-mosque is presented in Fig. 11.

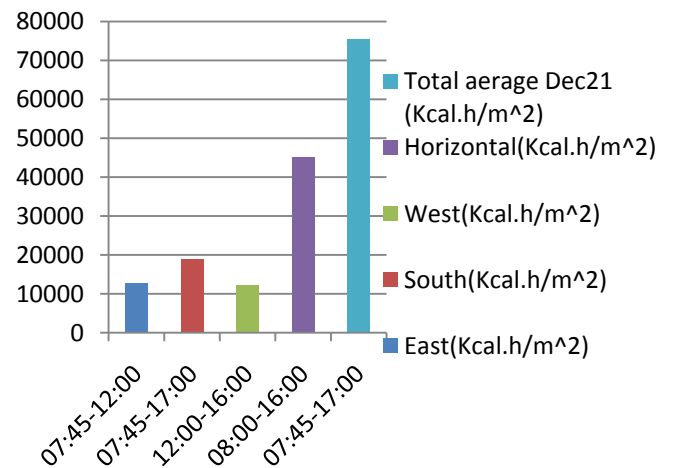


Fig. 11 Radiated solar energy on every surface

Fig. 11 shows the importance of surface and the aspects those are exposed on the rate of radiation beam. The proportion of surfaces in Blue-mosque is presented in Fig. 12.

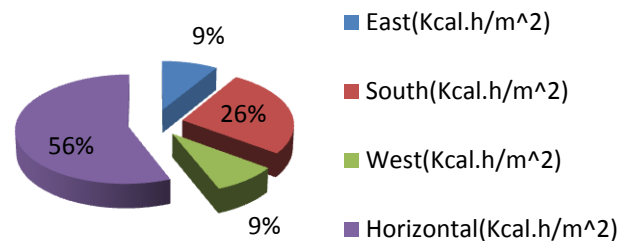


Fig. 12 Proportion of absorption on aspects

As it can be seen in Fig. 12, around  $7,54e+4$  (Kcal.h/m<sup>2</sup>) solar energy radiate on the Blue-mosque that its value is equal to around 9,43 (lit/h) energy of gasoil. In other word at least 75,44<sub>lit/day</sub> fossil energy is saved by the Blue-mosque.

## 7. CONCLUSION

Attention to the building formation in implement of passive solar energy is the first stage of sustainability. The barrier roles of elements faint their absorption abilities by overlapping specially in the frozen climate like Tabriz city.

The Blue-mosque of Tabriz is one of the Azeri (Turan-Iran) architectural masterworks that obey its climate. Analysing its shaded area that is the result of its elements asserts its climatic accordance. Implementing of the domes to cover the roofs was unavoidable in the past but their organization about locating and the innate trait of cubic forms let the Blue-mosque to compensate a portion of loosed area by the extra surfaces of domes that the shaded area on the roof decreased to 13% of its flat area in the critical situation of winter solstice. Also its southern aspect absorbs 4 times energy in compare of eastern or western aspects. Its architecture formation provide a position to decrease at least 9,43 (lit/h) gasoil.

It is recommended to form optimum geometries to obtain maximum rate of solar energy and also using indigenous materials according to the different climates to save or resist of thermal energy.

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## REFERENCES

- [1] Pirnia k. The Iranian architecture styles, Tehran, Souroush Danish, 2009, Vol 4, pp. 269-300.
- [2] Sutton GE, Roof spray for reduction in transmitted solar radiation, heating, Piping & Air conditioning (ASHVE Journal Section), September, 1950, pp. 131-137.
- [3] Stolwijik JAed. Energy Conservation Strategies in Buildings: Comfort, Acceptability, and Health, John B. Pierce Fndn. Of CT, inc, Yale Univ, New Haven, 1978, pp. 201-216.
- [4] Thomas B, Ross L Finney. Calculus and analytic geometry, 7 th ed, Tehran, Addison-Wesley, 1988, pp. 130-141.
- [5] Pirnia k. The Iranian architecture styles, 4th ed, Tehran: Souroush Danish, 2009, p. 288.
- [6] Kasmai M. Climatic & Architecture, 3 rd ed, Tehran; Khak, 2000, p. 150.
- [7] Donald Watson, Translation; Ghobadian Vahid Climatic design, Tehran University Publication, 1989, pp. 52-69.
- [8] Parker D, Wood A. The Tall Buildings Reference Book, Routledge, 2013.
- [9] Dian R, Si H. Double skin facades for office buildings, China Electric Power Press, 1991.
- [10] Aksamija A. Sustainable Facades: Design Methods for High-Performance Building Envelopes, Wiley, 2013.
- [11] Miyazaki T, Akisawa A, Kashiwagi T. Energy saving of office buildings by the use of semi-transparent solar cells for windows, Renewable Energy, 2005, No. 3, Vol. 30, pp. 281-304.
- [12] Ozdeniz M, Alibaba H. Thermal comfort of multiple-skin facades in warm-climate offices, Scientific Research and Essays, 2011, No. 19, Vol. 6, pp. 4065-4078.
- [13] URL1: <https://mr7uj.wordpress.com>, Accessed on 10 June, 2014.
- [14] Kasmai M. Climatic & Architecture, Tehran; Khak, 2000, Vol. 3, pp.140-170.
- [15] Thomas B, Ross L Finney. Calculus and analytic geometry, Tehran, Addison-Wesley, 1988, Vol 7, pp. 130-141.
- [16] Thomas B, Ross L Finney, 1988, pp. 130-141.
- [17] Ouria M. An overview of the historical and technical evolution of Azerbaijan timber architecture, Science Research, 2014, Vol. 3, pp. 22-26.
- [18] Lovel J. Building envelopes: an integrated approach (architecture briefs), Princeton Architectural Press, 2010.
- [19] Loren D. Enochson and Robert K. Programming and Analysis for Digital Time Series Data. U.S. Dept. of Defense, Shock and Vibration Info. Center, Otnes 1968, p. 142.
- [20] Leal V, Maldonado E, Erell E, Etzion Y. Modeling a reversible ventilated window for simulation within ESP-R-the solvent case, Proceedings of the 8th Int'l Conference on Building Performance Simulation, 2003, pp. 713-720.
- [21] Manz H, Frank Th. Thermal simulation of buildings with double-skin facades. Energy & Buildings, 2005, No. 37, Vol. 11, pp. 1114-1121.
- [22] Reda I, Andreas A. Solar position algorithm for solar radiation applications, Solar Energy, 2004, Vol. 76, pp. 577-589.

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